Neutrino oscillations in formal scattering theory

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Abstract

Scattering theory in the Gell-Mann–Goldberger formulation can be slightly extended in order to provide a quantum mechanical description of neutrino oscillations. Since the whole theory is based on the Hamiltonian description of quantum dynamics, it can be employed in the two translation-invariant forms of Hamiltonian dynamics that were identified by Dirac. We will explain the current status of the theory in the front form in which the evolution of states is traced in the parameter t + z with certain choice of the z-axis.

Please ignore
my difficulty with English.

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T2K Experiment as an example I

One can consider an example of T2K experiment in Japan.

$$t \sim 300\,\mathrm{km} = 1\,\mathrm{ms}$$

$$\tau_\pi \sim 1\,\mu\mathrm{s}$$

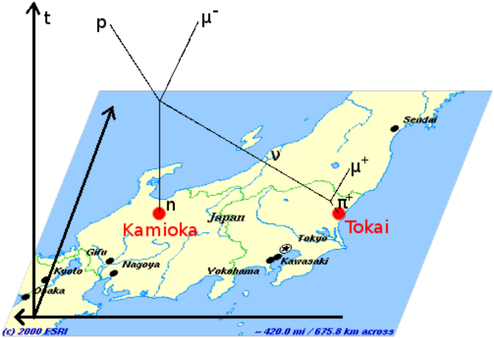

Figure: $\pi^+ n \to \mu^+ \mu^- p$

T2K Experiment as an example II

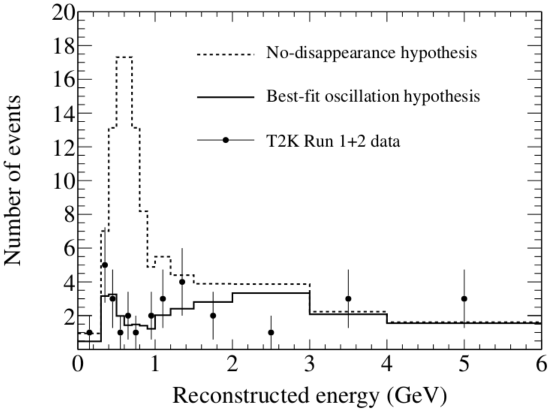

Figure: *First Muon-Neutrino Disappearance Study with an Off-Axis Beam* [1]. Number of detections as a function of neutrino reconstructed energy.

Standard interpretation of ν oscillation I

Bilenky and Pontecorvo derived a formula for so-called neutrino oscillations in 1977 [2] in the sense that the phase of a neutrino state changes with time and a combination of states with different energies involves a time-dependent phase difference.

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \xrightarrow{e^{-iH_0 t}|\nu_\alpha\rangle = \sum_i U_{\alpha i} e^{-iE_{\nu_i} t}|\nu_i\rangle} \left|\langle\nu_\beta|e^{-iH_0 t}|\nu_\alpha\rangle\right|^2 = P_{\alpha\to\beta}$$

Today, for example, it is said that the probability that a muon neutrino produced in Tokai will be detected in Kamioka, is smaller than 1 if there is the *neutrino oscillation effect*. The standard formula reads:

$$P_{\nu_\mu\to\nu_\mu} = \sum_{i,j}\left|U_{\mu i}U^\dagger_{j\mu}\right|^2 \mathrm{e}^{i\frac{m_i^2-m_j^2}{2E_\nu}L} \approx 1-\sin^2(2\theta_{23})\sin^2\left(\frac{\Delta m_{23}}{4E_\nu}L\right) .$$

Standard interpretation of ν oscillation II

Standard oscillation formula:

- does not have relativistic quantum-mechanical basis,
- assumes detection of neutrinos instead of muons,
- assumes the momenta $\vec{p}$ for different neutrinos,
- leaves unanswered questions about independence of physical E and $\vec{p}$ from the kind of neutrino.

$\Rightarrow$ All neutrinos cannot be on the mass shell.
$\Rightarrow$ QFT should be able to clarify the relativistic quantum picture.

Could one observe neutrino oscillation at rest?

Why ν oscillations on the light-front

Why the light-front:

- Good exercise for a beginner.
- New interpretation.
- New take on the vacuum.
- Preparation for studies of the neutrino mass origin (outside vacuum).

Remark: The light-front is the same for all observers moving in z-direction, i.e. it is the same in the *infinite momentum frame* and in the rest frame of neutrino.

The light-front formalism does not use Feynman diagrams. One needs a Hamiltonian dynamics description [3].

Hamiltonian description I

The incoming state $|\Psi_i\rangle$ is gradually built according to the formula [4]

$$\begin{aligned} |\Psi_i(x^+)\rangle &= \frac{\epsilon^-}{2}\int_{-\infty}^{0} d\mathcal{X}^+\, e^{\epsilon^-\mathcal{X}^+/2} e^{-iP^-(x^+-\mathcal{X}^+)/2}|\Phi_i(\mathcal{X}^+)\rangle \\ &= e^{-iP^-x^+/2}\,\frac{i\,\epsilon^-}{p_i^- - P^- + i\,\epsilon^-}|\phi_i\rangle\,. \end{aligned}$$

The transition rate of the evolving system to the final state $|\phi_f\rangle$ at x^+ is given by the derivative of the probability that the system is in state $|\phi_f\rangle$

$$P_{fi}(x^+) = \frac{d}{dt}\frac{|A(x^+)|^2}{||\Phi_f||^2||\Psi_i||^2}\,,$$

where the scattering amplitude is

$$A(x^+) = \langle\phi_f|e^{i\,p_f^-x^+/2}\cdot e^{-i\,P^-x^+/2}\frac{i\epsilon^-}{p_i^- - P^- + i\epsilon^-}|\phi_i\rangle\,.$$

Hamiltonian description II

The FF theory requires that the rates are evaluated with respect to x^+

$$\frac{\partial}{\partial x^+} = \frac{\partial t}{\partial x^+}\frac{\partial}{\partial t} + \frac{\partial z}{\partial x^+}\frac{\partial}{\partial z}\,.$$

In a standard long-base experiment

$$\frac{d}{dt}|A(x^+)|^2 = \frac{d}{dx^+/2}|A(x^+)|^2\,.$$

Hamiltonian description III

Following [5], the FF transition rate is obtained in the form

$$\frac{d}{dx^+/2}\,|A(x^+)|^2 \quad = \quad \frac{2\epsilon^-}{(p_f^- - p_i^-)^2 + (\epsilon^-)^2}|R_{fi}(x^+,\epsilon^-)|^2\,,$$

where $R_{fi}(x^+,\epsilon^-)$ is

$$R_{fi}(x^+,\epsilon) \quad = \quad \langle\phi_f|P_I^- e^{i(p_f^- - P_0^-)x^+/2}\frac{i\epsilon^-}{p_i^- - P^- + i\epsilon^-}|\phi_i\rangle\,.$$

Light-front calculation I

Lagrangian density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$

$$\begin{aligned}
\mathcal{L}_0 &= \sum_{\psi} \bar{\psi}(i\partial\!\!\!/ - m_\psi)\psi + \partial_\mu \pi^\dagger \partial^\mu \pi - m_\pi^2 \pi^\dagger \pi \\
\mathcal{L}_I &= \frac{G_F}{\sqrt{2}} \cos\vartheta_C \; \bar{\mu}\gamma^\alpha(1-\gamma_5)\nu_\mu \; \bar{p}\gamma_\alpha(1-g_A\gamma_5)n \\
&\quad - i\frac{F_\pi}{\sqrt{2}} \, \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \; \partial_\alpha \pi^\dagger + H.c. \; .
\end{aligned}$$

is used to derived

$$P^- = \int dx^- d^2x^\perp \mathcal{P}^- \, ,$$

where $\mathcal{P}^- = \frac{1}{2} T^{+-}$ and

$$T^{\mu\nu} = g^{\mu\alpha} \sum_\psi \frac{\partial \mathcal{L}}{\partial \partial^\alpha \psi} \partial^\nu \psi - g^{\mu\nu} \mathcal{L} \, .$$

Light-front calculation II

The constrain equation for fermions have a form

$$\psi^{(-)} = \psi_0^{(-)} + \psi_I^{(-)},$$

where $\psi_0^{(-)}$ is a free field and satisfies the equation

$$\psi_0^{(-)} = \frac{i\alpha^\perp\partial^\perp + \beta m_\psi}{i\partial^+}\psi_0^{(+)},$$

and we have to expand in powers of coupling constants g and f, so

$$\begin{aligned}
\mathcal{P}_0^- &= \partial^\perp\pi^\dagger\partial^\perp\pi + m_\pi^2\pi^\dagger\pi + \sum_{\psi_0}\bar{\psi}_0\gamma^+\frac{-\partial^{\perp\,2} + m_\psi^2}{2i\partial^+}\psi_0\,,\\
\mathcal{P}_1^- &= \left(if\partial^{\tilde{\alpha}}\pi^\dagger - gJ_{N0}^{\dagger\alpha}\right)J_{L0\alpha} + H.c.\,,\\
\mathcal{P}_2^- \ni \mathcal{P}_{gf}^- &= -igf\,\bar{\mu}_0\not{J}_{N0}(1-\gamma^5)\frac{\gamma^+}{2i\partial^+}\not{\partial}\pi^\dagger(1-\gamma^5)\mu_0\\
&\quad -igf\,\bar{\nu}_{\mu 0}\not{\partial}\pi^\dagger(1-\gamma^5)\frac{\gamma^+}{2i\partial^+}\not{J}_{N0}(1-\gamma^5) + H.c.\,.
\end{aligned}$$

Result

We obtain by substitution P^- to R_{fi}

$$R_{fi}(x^+,\epsilon^-) \quad = \quad \langle\phi_f|\left[P_1^- \frac{e^{i(p_i^- - P_0^-)x^+/2}}{p_i^- - P_0^- + i\epsilon^-} P_1^- + P_2^-\right]|\phi_i\rangle\,.$$

For $p_\nu^+ > 0 \Rightarrow$ exchange of neutrino.
For $p_\nu^+ < 0 \Rightarrow$ exchange of antyneutrino.
Seagull is present in the both cases.

For exchange of neutrino

$$\langle p\mu\bar{\mu}|P_g^- \frac{1}{p_i^- - P_0^- + i\epsilon} P_f^-|n\pi^+\rangle \quad \sim \quad \sum_i U_{i\mu} \frac{1}{p_\nu^+} \frac{\not{p}_{\nu_i} + m_{\nu_i}}{p_\nu^- - p_{\nu_i}^- + i\epsilon^-}\,,$$

$$\langle p\mu\bar{\mu}|P_{gf}^-|n\pi^+\rangle \quad \sim \quad \frac{\gamma^+}{2p_\nu^+}\,.$$

Conclusion

Numerical calculations have shown that

$$\langle p\mu\bar{\mu}|P_g^- \frac{1}{p_i^- - P_0^- + i\epsilon} P_f^- |n\pi^+\rangle \quad \gg \quad \langle p\mu\bar{\mu}|P_{gf}^- |n\pi^+\rangle \, .$$

Therefore, the first approximation to the transition rate is

$$P_{fi}(x^+ = 2L) \quad \propto \quad \left|\sum_i U_{i\mu} e^{-ip_{\nu_i}^- x^+/2}\right|^2 = \left|\sum_i U_{i\mu} e^{-i\,\frac{m_i^2}{p_\nu^+}\,\frac{2L}{2}}\right|^2 \, .$$

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Conclusion: We did it! It is possible to use the front-form to describe neutrino oscillation.

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